Prosimy cytować jako: Inż. Ap. Chem. 2011, 50, 2, 13-14

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Minimum energy of the intelligent grinding system

Introduction

The minimum energy principle states that all reactions, states of strain, states of stress, transformations and consequences of actions occurring within a machine, a technical system (during a grinding process) are always aimed at a reduction of free energy. Free energy as a part of total energy of the system that may be transformed into work in the process of constant temperature or that may be transformed into work in the process of constant temperature and pressure.

The aim of the work is to determine relationship between stresses and deformations and the minimum effective work (within the minimum energy) of the grinding process for the already stated principles of the intelligent grinding system and operational parameters – objectives of its operation.

Minimum energy

When operational parameters: product quality, energy efficiency of the process and harmlessness of interactions are limited to the effective work of the grinding process (pragmatically, logically and axiological useful), i.e. the state and transformation of internal stresses and external deformations – of the grain comminution, considering only one type of stress σ_x i.e. a linear state of stress, the following linear unit deformations of the analysed grains/granules may be described: toward x axis according to the first *Hooke's* law:

$$\varepsilon'_x = \frac{\sigma_x}{E}, \tag{1}$$

and towards y and z axes, lateral in relation to the direction of the force applied,

$$\varepsilon'_y = \varepsilon'_z = -\mu\frac{\sigma_x}{E}, \tag{2}$$

where:

E – longitudinal modulus of elasticity, constant for isotropic objects (within the elasticity limit),

μ – *Poisson* ratio (In the case of finite strains, this ratio is not a constant value, since it is a function of the strain value. The ratio may be considered constant for minor strains).

When considering only one type of strain σ_y and reasoning by analogy results in the following:

$$\varepsilon''_y = \frac{\sigma_y}{E}, \quad \varepsilon''_x = \varepsilon''_z = -\mu\frac{\sigma_y}{E} \tag{3}$$

And when considering only the effect of strains σ_z:

$$\varepsilon'''_z = \frac{\sigma_z}{E}, \quad \varepsilon'''_x = \varepsilon'''_y = -\mu\frac{\sigma_z}{E} \tag{4}$$

For normal strains analysed separately right angle strain (angles of non-dilatational strain) does not occur on walls of elementary grains/granules. Steady stress gives the opposite results, i.e. result in a change of the shape of the cuboid without a change in length of each edges; to be more precise, this type of strains are extremely small compared to angular strains. (The independence of angular strains or shear stresses of normal stresses may be also explained with the principle of symmetry and antisymetry used in structural analysis).

A pair of shear stress τ_{xy} - τ_{yx} results only in non-dilatational strain of walls parallel to xy plane, without an influence on other walls of cuboid. According to the second *Hooke's* law [2, 6, 9]:

$$\gamma'_{xy} = \frac{\tau_{xy}}{G}, \qquad \gamma'_{yz} = \gamma'_{zx} = 0 \tag{5}$$

where, as it is known:

$$G = \frac{E}{2(1+\mu)} \tag{6}$$

By analogy, pair $\tau_{yz} = \tau_{zy}$ gives:

$$\gamma''_{yz} = \frac{\tau_{yz}}{G}, \qquad \gamma''_{zx} = \gamma''_{xy} = 0 \tag{7}$$

pair $\tau_{xy} = \tau_{xz}$ gives:

$$\gamma'''_{zx} = \frac{\tau_{zx}}{G}, \qquad \gamma'''_{xy} = \gamma'''_{yz} = 0 \tag{8}$$

Based on that principle, with all components of the stress, expressions are obtained for components of the comminution state, strain at triaxial state of stress (9).

With $\mu = 0$, any component of the stress is directly proportional to the relevant component of strains. In the formulas (9) components of strains are expressed through components of stresses. In the grinding process, it is very often necessary to apply reverse relationships, i.e. those for which equations (9) are solved in relation to σ_x, σ_y, etc.

$$\left.\begin{aligned}
\varepsilon_x &= \frac{1}{E}[\sigma_x - \mu(\sigma_y+\sigma_z)], \ \gamma_{xy} = \frac{\tau_{xy}}{G},\\
\varepsilon_y &= \frac{1}{E}[\sigma_y - \mu(\sigma_z+\sigma_x)], \ \gamma_{yz} = \frac{\tau_{yz}}{G},\\
\varepsilon_z &= \frac{1}{E}[\sigma_z - \mu(\sigma_x+\sigma_y)], \ \gamma_{zx} = \frac{\tau_{zx}}{G}.
\end{aligned}\right\} \tag{9}$$

After conversion, the final results are obtained:

$$\left\{\begin{aligned}
\sigma_x &= 2G\left[\varepsilon_x + \frac{3\mu}{1-2\mu}\varepsilon_{śr}\right], \ \tau_{xy} = G\gamma_{xy},\\
\sigma_y &= 2G\left[\varepsilon_y + \frac{3\mu}{1-2\mu}\varepsilon_{śr}\right], \ \tau_{yz} = G\gamma_{yz},\\
\sigma_z &= 2G\left[\varepsilon_z + \frac{3\mu}{1-2\mu}\varepsilon_{śr}\right], \ \tau_{zx} = G\gamma_{zx},
\end{aligned}\right\} \tag{10}$$

Addition of left and right side of the first three relationships (9), gives the following:

$$\varepsilon_x + \varepsilon_y + \varepsilon_z = \frac{1-2\mu}{E}(\sigma_x + \sigma_y + \sigma_z) \tag{a}$$

Because:

$$\frac{1}{3}(\varepsilon_x+\varepsilon_y+\varepsilon_z) = \varepsilon_{śr} \quad \text{and} \quad \frac{1}{3}(\sigma_x+\sigma_y+\sigma_z) = \sigma_{śr}$$

these are so-called average strains and stresses, on the basis of (a):

$$\sigma_{śr} = \frac{E}{1-2\mu}\varepsilon_{śr} \tag{11}$$

Therefore, the average stress is proportional to the average strain of the grinding process. Because the sum of unit elongations in three mutually parallel directions given unit volumetric strain, i.e. $\varepsilon_{śr} = \Theta$ relationship (11) may also be presented as follows:

$$\sigma_{śr} = \frac{E}{3(1-2\mu)}\Theta, \tag{11a}$$

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i.e. *average strain at a given point is proportional to the unit volumetric strain in the neighbourhood of the point* [10].

Expressions (11) and (11a) are called a principle of an elastic change of comminution volume [9, 10]. Therefore, in practice volumetric strain, calculated on the basis of the following expression:

$$\Theta = \frac{3(1-2\mu)}{E}\sigma_{\acute{s}r}, \qquad (11b)$$

always disappears once its causes are removed.

When going back to formula (10) and taking away $\sigma_{\acute{s}r}$ from both sides of the first relationship, however, in relation to the right side expressing it with $\varepsilon_{\acute{s}r}$ under relationship (11). Then the following is obtained:

$$\sigma_x - \sigma_{\acute{s}r} = 2G\left[\varepsilon_x + \frac{3\mu}{1-2\mu}\varepsilon_{\acute{s}r}\right] - \frac{E}{1-2\mu}\varepsilon_{\acute{s}r}$$

When $E = 2(1+\mu)G$ is substituted, the following is obtained:

$$\sigma_x - \sigma_{\acute{s}r} = 2G(\varepsilon_x - \varepsilon_{\acute{s}r})$$

By analogy, the second and the third relationship are presented and a system of relationships (10) is obtained as follows:

$$\left.\begin{aligned} \sigma_x - \sigma_{\acute{s}r} &= 2G(\varepsilon_x - \varepsilon_{\acute{s}r}),\\ \sigma_y - \sigma_{\acute{s}r} &= 2G(\varepsilon_y - \varepsilon_{\acute{s}r}),\\ \sigma_z - \sigma_{\acute{s}r} &= 2G(\varepsilon_z - \varepsilon_{\acute{s}r}),\\ \tau_{xy} &= 2G\frac{1}{2}\gamma_{xy},\\ \tau_{yz} &= 2G\frac{1}{2}\gamma_{yz},\\ \tau_{zx} &= 2G\frac{1}{2}\gamma_{zx}. \end{aligned}\right\} \qquad (12)$$

The last statement (12) is surface, form, volume and first of all energy favourable in relation to the theory of comminution as regards plastic strain.

Summary

If the system of stresses $(\sigma_x - \sigma_{\acute{s}r}), (\sigma_y - \sigma_{\acute{s}r}), (\sigma_z - \sigma_{\acute{s}r}), \tau_{xy}, \tau_{yz}, \tau_{zx}$ may be called components of the stresses, corresponding to the change of shape with the effective work of comminution applied, and the system of strains $(\varepsilon_x - \varepsilon_{\acute{s}r}), (\varepsilon_y - \varepsilon_{\acute{s}r}), (\varepsilon_z - \varepsilon_{\acute{s}r}), \frac{1}{2}\gamma_{xy}, \frac{1}{2}\gamma_{yz}, \frac{1}{2}\gamma_{zx}$ – components of comminution strains, corresponding to the change of shape, then the general principle of the change of shape during the grinding process (12) may be expressed as follows: *components of stresses and strains for effective work (energy) of comminution, corresponding to the change of shape are mutually proportional, i.e. the former equal the latter multiplied by the doubled rigidity modulus*.

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