**Levels of intelligent grinding system**

**Introduction**

*The multi-level structure* means that each grinding machine system is made up of units, subunits, elements that are joined together to form elements of the next level etc. from molecules in the comminution sections to population of grinders, mills, crushers. Therefore, each entities is made up of sub-entities, those sub-entities are made up of their sub-entities etc.

The aim of the work is to empirically verify, for already specified principles of the intelligent grinding system and operational parameters with minimum energy and self-regulation of strains-aims of its operation, improvements at various levels of the intelligent grinding system and consequences of the same as regards elements and relations of the grinding space.

**Levels of grinding/comminution**

**Molecular level**

Molecules (of biological material and polymer material) of cross-sections, apart from tangential stresses that counter-balance the twisting moment, are subject to normal stresses that counter-balance each other. In the clamping section of the grain, the highest values of stress occur where supporting constrains prevent the complete deformation of the section.

Assuming axis *z* along the axis of grain/granule and the beginning of a coordinate system at its left end, the following assumed form of the expression for calculation of deformations, the comminution strain of any section at distance *z* from the clamped section [1–3]:

\[
\begin{align*}
\sigma_{r} &= \frac{d}{dz} - \frac{b^2}{G} E M_{xy} (1 - e^{-nz}) \\
\sigma_{z} &= \frac{E}{2(1 + \mu)} \\
\tau_{r} &= n a^2 - \frac{b^2}{a} M_{xy} e^{-nz}
\end{align*}
\]

or based on expression (1):

\[
\sigma_{r} = E \frac{\partial w}{\partial z} = n \frac{a^2 - b^2}{b} E M_{xy} (1 - e^{-nz})
\]

and substituting \( G = \frac{E}{2(1 + \mu)} \) the following is obtained:

\[
\sigma_{r} = n a^2 - \frac{b^2}{a} 2(1 + \mu) M_{xy} e^{-nz}
\]

The normal stresses (unevenly distributed throughout the section), in molecules of cross-sections of grains/granules, should change the tangential stresses \( \tau_{r} \) and \( \tau_{z} \) calculated for free torsion (for \( \sigma_{r} = 0 \)).
Obviously, it must also be assumed that tangential stresses $\tau_{xy}$ will occur.

For stresses $\tau_{xy}$ (this will also be the new assumption) the following expression is assumed [1]:

$$\tau_{xy} = k(a^2b^2 - b^2x - a^2y)e^{\frac{z}{c}}$$  \hspace{1cm} (3)

This expression fulfills the following conditions: for $z = \infty$, $\tau_{xy}$ equals zero, i.e. function $\tau_{xy}$ is related to function of free deformation (1).

Then, on the profile of the grain section, i.e. for points that fulfill equation $\frac{a^2}{c^2} + \frac{b^2}{c^2} = 1$ the following is obtained $\tau_{xy} = 0$ which for $\sigma_x = \sigma_y = 0$ corresponds to the case when the side surface of the grain is free from stresses. Constant $k$ depends on the twisting moment.

Differential equations for the balance at the molecular level, considering that $\sigma_x = \sigma_y = 0$ takes the following form:

$$\begin{align*}
\frac{\partial \sigma_{xy}}{\partial y} + \frac{\partial \sigma_{xy}}{\partial z} &= 0, \\
\frac{\partial \sigma_{zy}}{\partial x} + \frac{\partial \sigma_{zy}}{\partial z} &= 0, \\
\frac{\partial \sigma_{xz}}{\partial y} + \frac{\partial \sigma_{xz}}{\partial x} &= 0. 
\end{align*}$$  \hspace{1cm} (4)

Taking into consideration the expression assumed for $\tau_{xy}$, the first two equations (4), when integrated, result in the following:

$$\begin{align*}
\tau_{xy} &= -\frac{1}{n}e^{-4b}4ka^4y(a^2b^2 - b^2x^2 - a^2y^2) + \frac{2M}{\pi a b}y, \\
\tau_{zy} &= -\frac{1}{n}e^{-4b}4kb^4x(a^2b^2 - b^2x^2 - a^2y^2) + \frac{2M}{\pi a b}y.
\end{align*}$$  \hspace{1cm} (5)

The last two terms are any functions, independent of $z$ (they are integration constants) and they have the same form as respective expressions in the theory of free torsion of the elliptical section.

**Grain level**

Functions (5) fulfill the boundary conditions, because on the side surface of the grain, first terms of the formulas for $\tau_{xy}$ and $\tau_{zy}$ become equal to zero and second terms, taken together, fulfill the boundary condition (2). Replacing $\sigma_x$, $\sigma_y$, and $\tau_{xy}$ in equation (4) with values of expressions (2) and (5) a conclusion may be drawn that it will be fulfilled in terms of identity for:

$$k = \frac{n^4a^2 - b^2}{8n^2a b}(1 + \mu)M,$$  \hspace{1cm} (6)

Therefore, stress expressions (2), (3) and (5) fulfill the differential equations of the balance and the static boundary conditions, because both were used above. But unfortunately, due to the fact that expressions for deformation $\omega$ and for tangential stress $\tau_{xy}$ were assumed freely (however, in respect of the grain they correspond to the nature of the problem) above expressions (2), (3) and (5) do not fulfill conditions of inseparability of deformations.

The unit potential energy, for the grain/granule analysed $\sigma_x = \sigma_y = 0$ equals:

$$\varphi = \frac{1}{2G} \left[ \frac{1}{2(1 + \mu)} \sigma_x^2 + \tau_{xy}^2 + \tau_{zy}^2 + \tau_{xz}^2 \right].$$  \hspace{1cm} (7)

For the calculation of the potential energy of the whole grain, expression (7) must be integrated in respect of its volume, i.e.:

$$\Phi = \int_0^1 \int_0^1 \int_0^1 \varphi \, dx \, dy \, dz$$  \hspace{1cm} (8)

When functions (2), (3) and (5) are placed in expression (8), the following is obtained [3, 8]:

$$\Phi = \frac{1}{2G} \left[ (1 + \mu)^2 \frac{M^2}{\pi a b} \left( \frac{a^2b^2}{8} - \frac{n^2a^2b^2}{80} + \frac{n^2a^2b^2}{24} \left( \frac{1}{1 + \mu} \right) ight) + \frac{1}{2G} \frac{M^2(a^2 + b^2)}{\pi a b} \right].$$  \hspace{1cm} (9)

where $l$ means length of the grain/granule.

When expression (9) is differentiated in respect of $n$, and the derivative equated to zero, the following is obtained:

$$n^2 = \frac{1}{ab} \left[ \frac{a^2 + b^2}{ab} + \frac{\sqrt{a^2 + b^2}}{a^2b^2} + \frac{16}{1 + \mu} \right].$$

According to the conditions of the grain level, $n$ should be a real and positive number.

**Machine level**

Efficiency, the degree of comminution and effective work are directly related to strains and stresses. The target mass efficiency $Q_{0, b, c}$ of the grinding process, for a variable number of discs and the method of grain feeding was shown on the diagram (Fig. 1).

**Design conditions**

$W_{obs}$ of RWT-05:KZ multi-disc grinder were verified on the basis of genetic optimization for three kinds of fed materials (Tab. 1). Results of the analysis affected the improvements of the genetic procedure, motion parameters (speed, torque, power, acceleration, mass transition etc.) and indicated the preferred applications of technology for a specific material (Fig. 1, Tab. 1).

**Integrated levels**

Results of the rough theory of torsion and quasi-cutting of the comminution elliptical section may be applied to specific cases. Namely, for $a = b$ (circular section) $n = 1.45 \frac{L}{\mu}$ for such value of $n$, based on the formula (6): $k = 0$, so all terms of the expressions for stresses that contain $n$, equal zero.

The potential energy of the ground grain with the circular section is:

$$\Phi = \frac{1}{2G} \frac{M^2}{\pi a b}.$$

Tab. 1. Coefficient of efficiency of the grinding process for selected grain/granule materials

<table>
<thead>
<tr>
<th>Item</th>
<th>Material</th>
<th>Efficiency criterion/ kg·h⁻¹</th>
<th>Comminution degree criterion</th>
<th>Load criterion MPa⁻¹</th>
<th>Work criterion* N·m·J</th>
<th>Total $k$ coefficient</th>
<th>Optimum state</th>
</tr>
</thead>
<tbody>
<tr>
<td>I.</td>
<td>Grain</td>
<td>50.18</td>
<td>5.57</td>
<td>0.1149</td>
<td>29.9</td>
<td>960.234</td>
<td>Best (Optimus)</td>
</tr>
<tr>
<td>II.</td>
<td>PE-LD granules</td>
<td>64.80</td>
<td>3.68</td>
<td>0.0555</td>
<td>20.8</td>
<td>275.283</td>
<td>Good (Bonus)</td>
</tr>
<tr>
<td>III.</td>
<td>Granules with wood fibre</td>
<td>34.20</td>
<td>3.75</td>
<td>0.0833</td>
<td>27.7</td>
<td>295.927</td>
<td>Better (Melior)</td>
</tr>
</tbody>
</table>

*) for the purpose of calculations the direct value of effective work $L = L$, was used instead of the reverse of the same for a variable number of discs and a variable method of biomaterial feeding (in relation to the general mass efficiency).

In a special case, the model of system efficiency and product quality, considering dependence (1) [14] and (16a) [15] takes on the following form:

$$\lambda = \lambda (\alpha, \beta, (0.13 - 0.41n)M^2, \mu, \mu, P_{\mu}).$$  \hspace{1cm} (10)

The analysis concerned in the final stage of compression before quasi-cutting of the grain) the greatest, in terms of absolute value, normal stresses occur in the clamped section and these are by 65% greater than the greatest tangential stress for this section. When $a$ may be regarded as infinitely huge as compared to $b$ (granules of
The empirical verification of the objectives of the intelligent grinding system, described as four aspects of efficiency, confirmed the assumption made that loads generated within the grinding unit are transferred, as strains and stresses, onto elements and relations of the grinding area (machines, grains/granules). This statement relates to a specific design of the grinding system (RWT), molecules of biomaterials and polymer materials and grains of the same. The method of generating load influences the state of comminution strains and stresses. Even elastic stresses may result in comminution strains if the grain is subject to stirring and/or movement loads.

Due to a low speed of moved and stirred grains, the holding of the grain for the duration of quasi-cutting accompanied by torsion and potential acceleration of a particle of the grain as a result of a rebound after the comminution, it may be assumed that the second state, for the purpose of simplification, is characterized by a continuity of the motion trajectory and that the duration of movement with stirring and grinding equals the time of movement with stirring: \( t_{\text{RWT}} = t_{\text{RWT}} \).

RWT multi-disc grinders, with discs mounted coaxially in a vertical arrangement, guarantee an increasing speed of deformation, movement and stirring. Thanks to such properties of the motion, additional loads result in grinding of grains subject to plastic or even elastic stresses, also in clamping places between edges of the tool for quasi-cutting (with torsion).

**Summary**

The percentage values of target mass efficiency of the grinding system is 1.58 times greater than the same proportion of sides), the greatest normal stress in the clamped section is 1.58 times greater than \( t_{\text{max}} \) for the same section if the latter is calculated using the formula for free torsion.

**REFERENCES**


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