Velocity of micro- and nano-pieces in multi discs grinding

Introduction

The existing models of micro- and nano-biomass grinding or milling velocity processes is still incomplete and can be divided into two classes. The first class comprises all models suitable for analytical applications. These models allow derivation of closed form solutions involving algebra and calculus. The other class comprises discrete or iterative models that require numerical processing. The models belonging to the latter class will be referred to as "numerical". All but one available analytical models of velocity are restricted to ideal micro- and nano-milling processes with equal insert spacing, no insert throw, no tool eccentricity, no spindle tilt and in circular trajectories of cutting points. On the other hand, only one of the above limitations (i.e., circular paths of the cutting velocity applies to circular paths of the velocity processes is still incomplete and can be divided into two classes.

Aim of this work is the answer of question: which of the numerical models are capable of providing a specific pieces grinding solution for a chosen set of input parameters?

Velocity model

To solve the problem for the work purposes the simplifying assumptions were made (Fig. 1):
1. micro- and nano-pieces are moving on the smooth surface, in the hypothetical passage AB, length 2r, formed between two discs seated on the vertical axis CD and rotating with the constant angular velocity. Inside the passage made of the surfaces of discs, a spherical piece K with the mass m, is moving. At the initial instant the idealized micro-piece K is at rest in the distance OK = a from the axis of rotation.
2. micro- and nano-piece motion K will consist of relative motion which is straight – line motion along the passage and of transportation which is rotary motion around axis CD with the constant angular velocity.

![Fig. 1. A micro- and nano-piece of material in the inter-disc space of the grinder](image)

To determine the equation of the particle’s relative motion in the inter-disc space i.e. dependence between its distance x from the axis of rotation and time t, the forces reacting for a single piece were defined:
- force of gravity: \( Q = mg \),
- horizontal component reaction of the milled material: \( R_h \),
- vertical component reaction of the disc surfaces: \( N \),
- fictitious internal force of convection: \( B_v = mp_v = mx \omega^2 \)
- fictitious Coriolis force of inertia, \( B = mp = 2m\omega dx/dt \)

where \( dx/dt = w \) is the relative velocity of a micro-pieces.

Dynamic equation of relative motion of micro-particle is in the form
\[
mp_v = mg + N + R_h + B_v + B_c
\]
(1)

However projections of the equation to the Cartesian coordinates axes \((x, y, z)\):
\[
m \frac{d^2x}{dt^2} = m\omega^2 x
\]
(2)
\[
0 = 2m\omega \frac{dx}{dt} - R_h
\]
(3)
\[
0 = N - mg
\]
(4)

The equation leading to determination of dependence \( x = f(t) \), was obtained by the equation integration (2). The equation after simple conversion has the form
\[
\frac{d^2x}{dt^2} - \omega^2 x = 0
\]

It is quadratic linear differential equation with constant factors. The solution of the equation is function
\[
x = C_1 e^{\omega t} + C_2 e^{-\omega t}
\]
(5)

The integration constants were determined from the initial conditions, for \( t = 0, x = a \)
\[
w = \frac{dx}{dt} = 0
\]
(6)

The relative velocity of a micro-pieces is
\[
w = \frac{dx}{dt} = (-C_1 e^{\omega t} + C_2 e^{-\omega t}) \omega
\]
(7)

Replacing the condition (6) to the equations (5) and (7) we obtained \( C_1 = C_2 = a/2 \). The equation of a particle moving has the form
\[
x = \frac{a}{2} (e^{\omega t} + e^{-\omega t}) = a \cosh(\omega t)
\]
(8)

or (Fig. 1):
\[
x = C_1 \sinh \omega t + C_1 \cosh \omega t + \frac{B_v}{m\omega^2} (1 - \cosh \omega t)
\]
(9)

with:
\[
\frac{dx}{dt} = C_1 \omega \cosh \omega t + C_1 \omega \sinh \omega t - \frac{B_v}{m\omega^2} \omega \sin \omega t
\]
(10)

and for \( x = a \), \( \frac{dx}{dt} = 0 \), \( C_2 = a \), \( C_1 = 0 \)
\[
x = a \cosh \omega t + \frac{B_v}{m\omega^2} (1 - \cosh \omega t)
\]
(11)
\[
x = \left(a - \frac{B_v}{m\omega^2}\right) \cosh \omega t + \frac{B_v}{m\omega^2}
\]
(12)

for \( \varphi = \omega t \) and \( t = \frac{\varphi}{\omega} \) (Fig. 1):
\[
x = \left(a - \frac{B_v}{m\omega^2}\right) \cosh \varphi + \frac{B_v}{m\omega^2}
\]
(13)
and relative velocity:

$$w = \frac{dx}{dt} = \frac{a\omega}{2}(e^{aw} + e^{-aw}) = a \sinh \omega t$$

for disc field:

$$\frac{d^2 x}{dt^2} = \frac{(2\omega^2 \sqrt{x^2 + y^2} - \mu g)}{\sqrt{1 + \left(\frac{dy}{dx}\right)^2}} - \frac{2\omega y + \mu dy}{\sqrt{1 + \left(\frac{dy}{dx}\right)^2}} + 2\omega y$$

$$\frac{d^2 y}{dt^2} = \frac{(2\omega^2 \sqrt{x^2 + y^2} - \mu g)}{\sqrt{1 + \left(\frac{dy}{dx}\right)^2}} - \frac{2\omega x + \mu dx}{\sqrt{1 + \left(\frac{dy}{dx}\right)^2}}$$

were:

- $\mu$ – tribological, friction factor,
- $\frac{dy}{dx}$ – function of discs profile,
- $\nu_r$ – linear velocity of one micro-piece, as:

$$\nu_r = \sqrt{\frac{2\omega^2}{\sqrt{1 + \left(\frac{dy}{dx}\right)^2}}(r^2 - a^2) - 2g + \frac{\mu}{\sqrt{1 + \left(\frac{dy}{dx}\right)^2}}(r - a) + \nu_{to}^2}$$

As it results from the equation (3) and (14) the horizontal reaction of a piece of material to the walls of the hypothetical passage is:

$$R_x = 2m\omega \frac{dx}{dt} = mao^2(e^{aw} - e^{-aw}) = 2maao^2 \sinh \omega t$$

The time $t_o$ after which the micro- and nano-milled piece is going to leave the inter disc space was calculated from the equation (8) knowing that for $t = t_o$ it is $x = r_c$.

Thus the equation was obtained:

$$t_o = \frac{a}{2}(e^{aw} + e^{-aw})$$

After substitution $e^{aw} = a$ as a consequence of simple conversion such an equation was obtained:

$$aw^2 - 2\pi a + a = 0$$

which root is

$$u = \frac{\pi}{a}(r_c^2 - a^2)^{1/2}$$

Thus the dependence was obtained

$$e^{aw} = \frac{\pi}{a}(r_c^2 - a^2)^{1/2}$$

from the equation we received the searched time $t_o$ after which a micro-particle will leave the fictitious passage created between the surface of disc and hypothetical walls of milled material:

$$t_o = \frac{1}{2\omega} \ln \frac{\pi}{a}(r_c^2 - a^2)^{1/2}$$

Relative velocity $w_o$ of a milled micro-piece at the moment of leaving of inter – disc space was calculated directly from the equation (14), replacing $t = t_o$:

$$w_o = \frac{a\omega}{2}(e^{aw} - e^{-aw}) = \omega(r_c^2 - a^2)^{1/2}$$

Model discussion

The distance from the rotation axis $OK_c = a$ is a variable for the disc pack, it depends on the radius of hole spacing in the examined discs. A quasi-cutting pieces of material force acts on this radius. The situation is complicated when there are more than one row of holes in the discs. In such a situation it is assumed that the distance $OK_c$ is related to the furthest – from the rotation center – milling edge.

The modeling methodology and equations of velocity grinding presented in this paper are based upon the results obtained by several researchers over a period of the last years. This methodology represents a unified approach, in which the essential features of previously developed analytic and numerical models are integrated. The model given by Eqs. (18) and (19) accounts for the following features and phenomena characteristic of micro- and nano-milling:

- tool eccentricity,
- insert throw,
- uneven insert spacing,
- spindle tilt, either steady or variable,
- spindle error motion,
- changes in the effective disc geometry due to machining parameters, type of operation (up- or down-milling) and spindle tilt,
- deflections of the machine, disc and work piece caused by grinding forces.

Conclusions

The proposed model results from a systematic approach to the micro- and nano-grinding process. It facilitates, therefore, an orderly derivation of versatile models, most appropriate for the particular investigated processes. The development of models begins with the selection of coordinate systems shown in Fig. 1. The same general form of final equations represents models of varying complexity.

Another important attribute of the employed systematic approach is the separation of various features and phenomena related to disc micro- and nano-grinding.

The systematic approach underlying the proposed methodology assures the coherence and compatibility of various derived models. Thus, these models can be developed incrementally, by increasing their sophistication until the desirable level of agreement with the actual process, or performance to cost ratio are achieved.

A motivation to derive the presented model came from the research of indirect, on-line tool condition monitoring methods. It was observed that the measured grinding forces did not correlate well with forces predicted by the available models. Discrepancies between the measured and predicted forces were particularly strong at small thicknesses, i.e., during entries and exits of grinding inserts from the machined material.

REFERENCES


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